Exercise 1.1

1. Write inductive definitions of the following sets. Write each definition in all three styles (top-down, bottom-up, rules of inference)
2. Show the derivation of some sample elements of each set.

**{3n+2| n € N}**

Top-down

1. A natural number n is in S if and only if either n=2S or n-3S
2. n=2 or n-3S N={2,5,8,11,14,17,20,23,…}
3. Running 5 through the sequence: 5 does not equal 2. So 5-3=2 and 2 does equal n which is in the sequence.

Running 6 through the sequence: 6 does not equal 2. So 6-3=3 and 3 does not equal 2. So run it a third time 3-3=0 and zero is not a natural number so ultimately 6 is not in the sequence.

Bottom-up

1. Define the set S to be the smallest set contained in N and satisfying the following two properties: 2S and if nS, then n+3S.
2. Running 2 through the sequence because we already know that 2 is in the sequence we get 2+3=5. Now 2 and 5 are in the sequence. So we will then add 5+3=8. Now 2,5,8, are in the sequence. (So on and so forth)

Rules of inference:

2S nS/(n+3)S

**{2n+3m + 1| n,mN}**

Top-down

1. A natural number n is in S if and only if n=1 or n-2S or n-3S
2. 1N, n=1 or n-2S or n-3S N={1,3,4,5,6,7,…}
3. Running 4 through the sequence: 4 is not 1, so 4-2=2 and 2 is not 1. So instead 4-3 which does indeed equal 1. So 1 and 4 are in the sequence.

Running 2 through the sequence: 2 does not equal 1, so 2-2=0 and that does not equal 1 either, so 2-3=-1 and neither does that. So 2 is not in the sequence.

Bottom-up

1. Define the set S to be the smallest set contained in N and satisfying the following two properties: 1S and if nS, then n+2S, if nS, then n+3S
2. Running 3 through the sequence, 3 does not equal 1, so 3+2=5. Thus 5 is in the sequence.

Rules of inference

1S nS/(n+2)S nS/(n+3)S

**{(n, 2n+1)|nN}**

Top-down

1. A natural number (n,m) is in S if and only if (n,m)=(0,1) or (n-1,m-2)
2. (n,m)S, (n,m)=(0,1) or (n-1,m-2) N={(0,1), (1,3),(2,5),(3,7),(4,9),…}
3. Running (2,5) through the sequence: (2,5) does not correspond with the coordinates (0,1) so 2-1=1, and 5-2=3. These new coordinates (1,3) do not correspond with (0,1), so 1-1=0, and 3-2=1. (0,1) does match (0,1) so coordinates (2,5) and (1,3) are both in the sequence.

Bottom-up

1. Define the set S to be the smallest set contained in N and satisfying the following two properties: (0,1)S if (n,m)S then (n+1, m+2) S
2. Running (1,3), it does not match (0,1). So 1+1=2 and 3+2=5. Thus (2,5) is in the sequence.

Rules of inference

(0,1)S (n,m)S/(n+1,m+2)S

**{(n, n^2)|nN}** \*Not mentioning the squaring rules

Top-down

1. A natural number (n,m) is in S if and only if (n,m)=(0,0) or (n-1, m-2n+1)S
2. (n,m)S, (n,m)=(0,0) or (n-1, m-2n+1)S N={(0,0),(1,1),(2,4),(3,9),(4,16),…}
3. Running (2,4) into the sequence, (2,4) does not equal coordinates (0,0). So 2-1=1 and 4-4+1=1. But the new coordinate (1,1) also does not equal (0,0). So 1-1=0 and 1-2+1=0. (0,0) does equal (0,0) so both (2,4) and (1,1) are in the sequence.

Bottom-up

1. Define the set S to be the smallest set contained in N and satisfying the following two properties: (0,0)S if (n,m)S then (n+1, m+2n+1)S
2. Running (2,4) into the sequence, (2,4) does not equal coordinates (0,0). So 2+1=3 and 4+4+1=9. Thus (3,9) is in the sequence.

Rules of inference

(0,0)S (n,m)S/(n+1,m+2n+1)S